SM121A-3003 EXAM THREE Friday, October 25, 2002

Work the following problems on the paper provided. I am much less interested in the answers to the questions than in the processes you use to find the answers. You may use your calculator but NOT your textbook. When you use your calculator, please make it clear to me what you asked it to do for you.

1. (30 points) Suppose that F and G are functions and that

   $F(2) = 3$ $F'(2) = 1.7$
   $F(3) = 1$ $F'(3) = 0.3$
   $G(1) = 4$ $G'(1) = -0.8$
   $G(3) = 2$ $G'(3) = 1.5$

   Calculate the following numbers, or indicate that you have insufficient information.
   a) The derivative of F+G when x = 3.
   b) The derivative of FG when x = 3.
   c) The derivative of G/F when x = 3.
   d) The derivative of the composition $F \circ G$ when x = 3.
   e) The derivative of the composition $G \circ F$ when x = 3.

2. (20) The equations below are parametric equations for an ellipse. Find an equation for the line tangent to the ellipse at the point where $t = \pi/6$.

   $$x = 5\sin(t) + 9 \qquad y = 3\cos(t) - 9 \qquad -\infty < t < \infty$$

3. (20) The equation below defines an ellipse. Find an equation for the line tangent to the ellipse at the point (-1, 3).

   $$x^2 + 3xy + 3y^2 = 19$$

4. (30) Suppose F is a function with the property that $F'(x) = (1 + F(x)^3)^{1/2}$ for every number x. (Do <u>not</u> try to find a formula for the function F.)
   a) Suppose F(1) = 2. Find a linear approximation to F near the point (1, 2).
   b) Use your linear approximation to estimate F(0.98).
   c) The function F has an inverse function G. What's G(2)?
   d) Use the fact that F(G(x)) = x for every number x to calculate a formula for G'(x). (This is the method we used to compute the derivatives of ln, arctan, and arcsin. Take the formula F(G(x)) = x and differentiate with respect to x.)
   e) What is G'(2)? (It's possible to answer this question even if you can't do part d. To do that, you need to see how the graphs of F and G are related.)
   f) Use this information to estimate G(2.1).